Chapter 5 Review

1. Write a recursive equation that represents the monthly account balance given that you take out a loan for $10,000 at a 6% APR and you want to pay it off over 5 years. Let x represent the value of your monthly payments.

$$f(n) = \begin{cases} 10,000 & if\ n = 0 \\ 1.005 \cdot f(n-1) - x & if\ n > 0 \end{cases}$$

2. Write a closed form equation for #1. Then use this to determine the value of your monthly payment, x.

$$f(n) = 10000 \cdot 1.005^n + \frac{-x \cdot (1 - 1.005^n)}{1 - 1.005}$$

Solve $0 = 10000 \cdot 1.005^{60} + \frac{-x \cdot (1 - 1.005^{60})}{1 - 1.005}$ to get x = $193.33

3. Write a recursive definition that agrees with the values in the table below

n	g(n)
0	3
1	4
2	7
3	11
4	18
5	29
6	47

$$g(n) = \begin{cases} 3, & n = 0 \\ 4, & n = 1 \\ g(n-1) + g(n-2) & if\ n > 1 \end{cases}$$

4. Find a recursively defined function that agrees with $f(n) = 3n^2 + n + 2$ for all nonnegative integers n.

$$f(n) = \begin{cases} 2 & if\ n = 0 \\ f(n-1) + 6n - 2 & if\ n > 0 \end{cases}$$

5. Briefly describe two different options that you can use to determine a polynomial function when you are given an input-output table. Then use one of those methods to determine a closed form equation for the table below. Write the closed form equation in descending order. (3 points)

Input x	Output g(x)
0	5
1	15
2	31
3	41
4	33
5	-5

In order to write a closed form equation, you could create a difference table to determine the degree of the function. A difference table that has a 2nd change that is constant will be a 2nd degree equation. A table that possesses a 3rd change that is constant will be a 3rd degree equation..etc. After determining the degree, you could then create a system of equations that would allow you to determine the coefficients by plugging in corresponding input and output values.

Your other option would be to use the top row of values from the change table and the connection to Pascal's Triangle.

$$5\binom{n}{0} + 10\binom{n}{1} + 6\binom{n}{2} - 12\binom{n}{3}$$

Answer: $g(x) = -2x^3 + 9x^2 + 3x + 5$

6. Find a closed form equation for the recursive equation

$$g(n) = \begin{cases} 4 \; if \; n = 0 \\ 11 \; if \; n = 1 \\ 7 \cdot g(n-1) - 10 \cdot g(n-2) if \; n > 1 \end{cases}.$$

A change table has been provided

n	g(n)	Δ	Δ2	Δ3
0	4	7	19	67
1	11	26	86	326
2	37	112	412	1612
3	149	524	2024	8024
4	673	2548	10048	40048
5	3221	12596	50096	200096
6	15817	62692	250192	1000192
7	78509	312884	1250384	5000384
8	391393	1563268	6250768	25000768
9	1954661	7814036	31251536	
10	9768697	39065572		
11	48834269			

$$g(n) = A \cdot 2^n + B \cdot 5^n$$

Set up a System of Equations

$$4 = A \cdot 2^0 + B \cdot 5^0$$

$$11 = A \cdot 2^1 + B \cdot 5^1$$

$$B = 1 \; and \; A = 3$$

$$g(n) = 3 \cdot 2^n + 1 \cdot 5^n$$