Name: ___________
Period: ___________

5.9 Newton's Formula and Mahler Polynomials

x	f(x)	Δ	$Δ^2$	$Δ^3$
0	5	10	6	−12
1	15	16	−6	−12
2	31	10	−18	−12
3	41	−8	−30	
4	33	−38		
5	−5			

A difference table is shown above for a polynomial function of third degree. Based on previous experiments, you discovered you could determine an output for any input using the first row of the difference table.

Example:

$$f(4)=5\cdot\binom{4}{0}+10\cdot\binom{4}{1}+6\cdot\binom{4}{2}+-12\cdot\binom{4}{3}+0\cdot\binom{4}{4}$$
$$f(5)=5\cdot\binom{5}{0}+10\cdot\binom{5}{1}+6\cdot\binom{5}{2}+-12\cdot\binom{5}{3}+0\cdot\binom{5}{4}+0\cdot\binom{5}{5}$$
$$f(6)=5\cdot\binom{6}{0}+10\cdot\binom{6}{1}+6\cdot\binom{6}{2}+-12\cdot\binom{6}{3}+0\cdot\binom{6}{4}+0\cdot\binom{6}{5}+0\cdot\binom{6}{6}$$

This pattern leads to the idea that for any input x, your output must be as follows.

$$f(x)=5\cdot\binom{x}{0}+10\cdot\binom{x}{1}+6\cdot\binom{x}{2}+-12\cdot\binom{x}{3}+0\cdot\binom{x}{4}+0\cdot\binom{x}{5}+0\cdot\binom{x}{6}\text{.........}0\binom{x}{x}$$

The above equation allows us to generate a closed form equation for f(x) provided that , we are able to determine polynomial expressions for ${}_xC_0$, ${}_xC_1$, ${}_xC_2$, and ${}_xC_3$.

Determine the values for these combinations on the back side of the paper.

Name: ___________
Period: ___________

1. Determine the polynomial expression for each combination. Show work, or write explanations on how each expression was determined. (It may be helpful to look at examples that deal with all numerical values to determine a pattern that will occur regardless of the value of the x variable shown below.)

${}_xC_0 =$

${}_xC_1 =$

${}_xC_2 =$

${}_xC_3 =$

2. After determining the values of the above combinations, insert them into the formula from the previous page $f(x) = 5 \cdot \begin{pmatrix} x \\ 0 \end{pmatrix} + 10 \cdot \begin{pmatrix} x \\ 1 \end{pmatrix} + 6 \cdot \begin{pmatrix} x \\ 2 \end{pmatrix} + -12 \cdot \begin{pmatrix} x \\ 3 \end{pmatrix}$ and expand the expression in order to write it as a polynomial function in descending order.

3. What is the relationship between the leading coefficient and the Δ^3 value for every 3rd degree polynomial equation?

4. Look closely at the ${}_xC_3$ expression you computed above and explain why the relationship from question 3 exists.

5. Determine the relationship between the leading coefficient and Δ^4 value for any 4th degree polynomial equation and explain how you determined your answer.