5.12 Part B
Geometric Sequences and Series Review

Goal: Become more familiar with geometric sequences and be able to compute the sum of a geometric series.

What is a geometric sequence?

*A geometric sequence goes from one term to the next by always multiplying (or dividing) by the same **rate**. So 1, 2, 4, 8, 16,... and 81, 27, 9, 3, 1, 1/3,... are geometric, since you multiply by 2 and divide by 3, respectively, at each step.*

Important concepts to consider:

Rate: The value that is used to multiply one term to generate the next consecutive term.
Initial value: The first term of the sequence often symbolized as a_1
Number of terms: How many terms exist in the sequence.

Example #1:

Determine the sum of the first 10 terms of the series S. The first 7 terms have been shown. S = 3, 6, 12, 24, 48, 96, 192….

Method 1: Use the summation command on your calculator to find the sum of the first 10 terms of S.
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To use this aspect, you must first express your sequence as a summation.

$$S = \sum_{k=0}^{9} 3 \cdot 2^k$$

To enter the summation into the TI 84, use the following commands: sum(seq(3(2^x),X,0,9,1))

Note:
sum(is found in: [2nd][Stat](List) > Math > 5:sum(
seq(is found in: [2nd][Stat](List) > Ops > 5:seq(

Method #2 Use a formula to find the sum of n terms.

$S = \frac{a_1(1-r^n)}{1-r}$ Where a_1 is equal to the initial term and r represents the rate of increase or decrease.

$$S = \frac{3(1-2^{10})}{1-2} = 3069$$

1. Prove that the formula $S = \frac{a_1(1-r^n)}{1-r}$ is true.

2. If $S = \frac{a_1(1-r^n)}{1-r}$, explain why the $\lim_{n\to\infty} \frac{a_1(1-r^n)}{1-r} = \frac{a_1}{1-r}$

3. $\lim_{n\to\infty} \frac{a_1(1-r^n)}{1-r} = \frac{a_1}{1-r}$ is only true provided that -1 < r < 1. Why?

Answer the following questions that relate to geometric sequences and series.

1. Find the 7^{th} term of the sequence 40, 4, .4, …..

2. Represent the above sequence as a summation. Then find the value of the sum of the first 7 terms.

3. Find the sum of the series 4, 2, 1, $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$......given that it is an infinite series.

   Show all work.

4. Express the sum of the geometric series 3, 6, 12, 24, ……as a summation. Then explain why it is impossible to calculate the sum of the infinite geometric series.

5. Express the series 3, 6, 12, 24…..as a recursive equation.