

5.12 Recurrences

Consider the function r that satisfies the recurrence $r(n) = 7 \cdot r(n-1) - 10 \cdot r(n-2)$. A table of values generated with Excel has been listed for each of the following base cases.

$r(0) = 1$ $r(1) = 5$ Table 1	$r(0) = 1$ $r(1) = 2$ Table 2	$r(0) = 2$ $r(1) = 7$ Table 3	$r(0) = 0$ $r(1) = 3$ Table 4																																																																																
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1. The first two base cases for Table 1 and Table 2 were strategically chosen by the book because they generate an exponential pattern. In particular focus on the $r(1)$ values. What do you notice about the relationship that exists between the 5 and 2 when compared to values of 7 and 10 that exist in the equation $r(n) = 7 \cdot r(n-1) - 10 \cdot r(n-2)$?

2. Determine a closed form equation that fits Table 1.

3. Determine a closed form equation that fits Table 2.

4. Compare the outputs from Table 3 to Table 1 and Table 2 simultaneously. What is the relationship between the tables?

5. Determine a closed form equation for Table 3.

6. Compare the outputs from Table 4 to Table 1 and Table 2 simultaneously. What is the relationship between the tables?

7. Determine a closed form equation for Table 4.