5.12 Recurrences PART A

Goal: Given any polynomial function, write it as a recursive equation.

Example: Express $f(x)= 3x^2 - 2x + 1$ in recursive format.

As in past cases, it is helpful to write out a difference table as well as equations that show the recursive relationship between each successive output.

x	f(x)	Δ	Δ2
0	1	1	6
1	2	7	6
2	9	13	6
3	22	19	6
4	41	25	6
5	66	31	6
6	97	37	6
7	134	43	6
8	177	49	6
9	226	55	
10	281		

$f(0) = 1$

$f(1) = f(0) + 1$

$f(2) = f(1) + 7$

$f(3) = f(2) + 13$

$f(4) = f(3) + 19$

$\Downarrow$

$\Downarrow$

$f(n) = f(n-1) + ??$

To complete the recursive equation, it is necessary to find a way to express the values 1, 7, 13, 19….etc. The easiest way is to express these in relation to the input value.

In other words, we must find a closed-form definition that represents the change column.

x	Δf	Δ
0	1	6
1	7	6
2	13	6
3	19	6
4	25	6

Based on the table you see a first degree pattern emerge. It appears as if the equation should be Δ(x) = 6x +1.

Δ(0) = 1
Δ(1) = 7
Δ(2) = 13
Δ(3) = 19

This fits the change table, but if you view the above recursive form, you will notice a discrepancy.

A student responds with the answer $f(n) = \begin{cases} 1 \text{ if } n = 0 \\ f(n-1) + 6n + 1 \text{ if } n > 0 \end{cases}$ as a recursive equation that they believe fits the change table.

This is incorrect. Explain the error that was made and find the correct recursive equation.

Find recursive equations for the following functions given the change tables.

$f(n) = 2x^3 - 4x^2 + 5x - 3$

x	f(x)	Δ	Δ2	Δ3
0	-3	3	4	12
1	0	7	16	12
2	7	23	28	12
3	30	51	40	12
4	81	91	52	12
5	172	143	64	12
6	315	207	76	12
7	522	283	88	12
8	805	371	100	
9	1176	471		
10	1647			

$g(n) = x^4 + x^3 - 3x^2 + 6x - 2$

x	f(x)	Δ	Δ2	Δ3	Δ4
0	-2	5	14	42	24
1	3	19	56	66	24
2	22	75	122	90	24
3	97	197	212	114	24
4	294	409	326	138	24
5	703	735	464	162	24
6	1438	1199	626	186	24
7	2637	1825	812	210	
8	4462	2637	1022		
9	7099	3659			
10	10758				