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Since the logarithmic function and the exponential function are inverses of each other, both of their compositions yield the identity function. Let $f(x) = \log_a x$ and $g(x) = a^x$. For $f(x)$ and $g(x)$ to be inverses, it must be true that $f(g(x)) = x$ and $g(f(x)) = x$.

$$\begin{aligned} f(g(x)) &\stackrel{?}{=} x \\ f(a^x) &\stackrel{?}{=} x \\ \log_a a^x &\stackrel{?}{=} x \\ x &= x \end{aligned} \qquad \begin{aligned} g(f(x)) &\stackrel{?}{=} x \\ g(\log_a x) &\stackrel{?}{=} x \\ a^{\log_a x} &\stackrel{?}{=} x \\ x &= x \end{aligned}$$

The properties of logarithms can be derived from the properties of exponents.

Properties of Logarithms

Suppose m and n are positive numbers, b is a positive number other than 1, and p is any real number. Then the following properties hold.

Property	**Definition**	**Example**
Product	$\log_b mn = \log_b m + \log_b n$	$\log_3 9x = \log_3 9 + \log_3 x$
Quotient	$\log_b \frac{m}{n} = \log_b m - \log_b n$	$\log_{\frac{1}{4}} \frac{4}{5} = \log_{\frac{1}{4}} 4 - \log_{\frac{1}{4}} 5$
Power	$\log_b m^p = p \cdot \log_b m$	$\log_2 8^x = x \cdot \log_2 8$
Equality	If $\log_b m = \log_b n$, then $m = n$.	$\log_8 (3x - 4) = \log_8 (5x + 2)$ so, $3x - 4 = 5x + 2$

Each of these properties can be verified using the properties of exponents. For example, suppose we want to prove the Product Property. Let $x = \log_b m$ and $y = \log_b n$. Then by definition $b^x = m$ and $b^y = n$.

$$\begin{aligned} \log_b mn &= \log_b (b^x \cdot b^y) && b^x = m,\ b^y = n \\ &= \log_b (b^{x+y}) && \textit{Product Property of Exponents} \\ &= x + y && \textit{Definition of logarithm} \\ &= \log_b m + \log_b n && \textit{Substitution} \end{aligned}$$