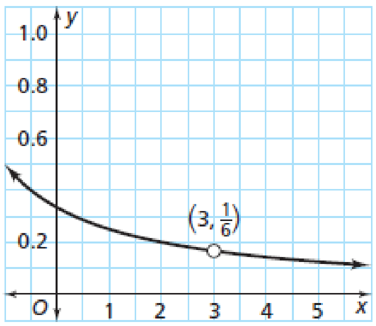
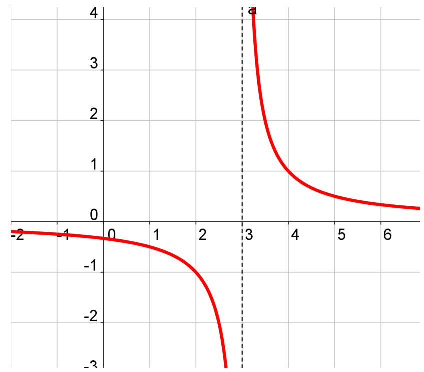
3. 6 Rational Functions

Goal: Become more familiar with rational functions and discontinuities.

All rational functions will have discontinuities that will result in one of two types of breaks in the graphs: **Point Discontinuity** (Hole) or **Infinite Discontinuity** (Vertical Asymptote).



Point Discontinuity at x = 3



Infinite discontinuity at x = 3

Discontinuities are expressed in terms of x values, or the input.

The first step in working with rational functions is

- **Determining where a discontinuity exists**

$f(x) = \frac{1}{x-3}$	Rational functions possess a discontinuity when they are UNDEFINED. Functions are undefined when the **denominator is equal to zero** **f(x) has a discontinuity at x = 3.** Rational functions can be described by discussing their domains. For the function at the left you may describe the Domain as follows • Domain of f(x) is all Real numbers BUT x = 3 • Domain is $\{x \mid x \neq 3\}$ (This is read the set of all x values such that x is not equal to three)
$g(x) = \frac{1}{x^2 + 9x + 14}$	Find the discontinuities of g(x) and discuss the Domain of g(x) 1. Discontinuities: 2. Domain:

The 2nd step in working with rational functions is

- **Determining if the discontinuity is a vertical asymptote or a hole**

To determine the type of discontinuity without viewing the graph, it is useful to work with the limits of the function as they approach the discontinuity.

$f(x) = \frac{1}{x-3}$	Question: Is the discontinuity at x = 3 a point discontinuity or an infinite discontinuity? ***Consider the limits as x approaches 3 from the left and right side. If these limits are equal to the same numeric value, that should indicate a point discontinuity. If the limits approach ∞ or -∞, then there will likely be an asymptote at that input.*** **Determine the limit as x approaches 3 from the left** Find the $\lim_{x \to 3^-} f(x)$ by plugging in inputs that get closer and closer to 3 from the left. 3. f(2.9) = ______ f(2.99) = _____ f(2.999) = ________ 4. Based on your previous answers, what is the $\lim_{x \to 3^-} f(x)$? **Determine the limit as x approaches 3 from the right** Now consider the limit as x approaches 3 from the right side $\lim_{x \to 3^+} f(x)$. To determine this, plug in inputs that get closer and closer to 3 from the right. 5. f(3.1) = ______ f(3.01) = _____ f(3.001) = ________ 6. Based on your previous answers, what is the $\lim_{x \to 3^+} f(x)$? Based on your answers to question #4 and #6, you should realize that this is an **<u>infinite discontinuity</u>** because the limits from the left and right approach negative and positive infinity. View the graph and take note of how your results correlate to the graph.

Determine where $f(x)=\dfrac{x^2-6x+8}{x-4}$ is undefined and what type of discontinuity exists.

Step #1: Determine where it is undefined.

Answer: x = 4

Step #2: Use limits to determine the type of discontinuity

Determine the limit as x approaches 4 from the left

Find the $\lim_{x\to 4^-} f(x)$ by plugging in inputs that get closer and closer to 4 from the left.

7. f(3.9) = ______ f(3.99) = _____ f(3.999) = ________
8. Based on your previous answers, what is the $\lim_{x\to 4^-} f(x)$?

Determine the limit as x approaches 4 from the right $\lim_{x\to 4^+} f(x)$. To determine this, plug in inputs that get closer and closer to 4 from the right.

9. f(4.1) = ______ f(4.01) = _____ f(4.001) = ________
10. Based on your previous answers, what is the $\lim_{x\to 4^+} f(x)$?

Based on your answers to questions #8 and #9, you should realize that this is a **point discontinuity** because $\lim_{x\to 4^-} f(x)=\lim_{x\to 4^+} f(x)$. Both of which equal a numeric value of 2.

What is a quicker way to determine the type of discontinuity that will exist?

The approach of plugging in values to determine the limit is often tedious and time consuming. A quick way to determine if a discontinuity will be a hole or asymptote is to decide if a rational function can be simplified.

$$f(x)=\frac{x^2-6x+8}{x-4}=\frac{(x-2)(x-4)}{x-4}=x-2$$

This displays that the rational function f(x) will behave exactly like the linear function of x-2. There is a point discontinuity at x = 4. NOTICE that this occurs when you have a common factor in the numerator and denominator that cancels.

HOLES OCCUR WHEN THE NUMERATOR AND DENOMINATOR HAVE COMMON FACTORS

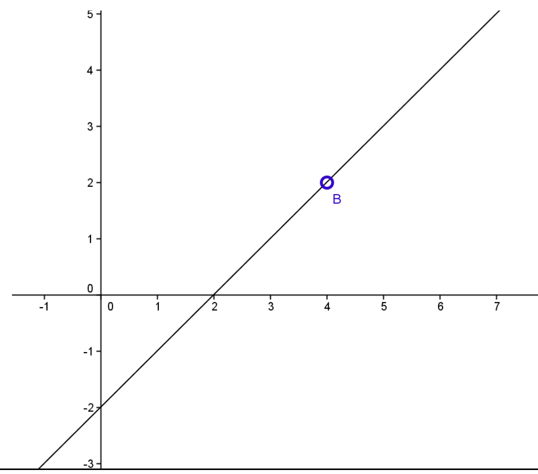
Graphing $f(x) = \dfrac{x^2 - 6x + 8}{x - 4}$.

The graph of f(x) may seem very intimidating, but if you simplify the function first, the graph is very straight forward.

$$f(x) = \frac{x^2 - 6x + 8}{x - 4} = \frac{(x-2)(x-4)}{x-4} = x - 2$$

Therefore f(x) = x -2 which can be represented as a linear equation with a hole.

The hole exists at x = 4 because that is where our original function f(x) was undefined. It has a coordinate value of (4,2) because f(x) = x – 2 and f(4) = 4 -2 = 2.



Determine where the discontinuities will occur in the following functions, then classify them as a point or infinite discontinuity. Finally discuss the domain of each function.

11. $f(x) = \dfrac{x^2 - 6x + 9}{x^2 - x - 6}$

12. $f(x) = \dfrac{x^2 - 1}{x^2 - 2x + 1}$

13. $f(x) = \dfrac{(x-1)(x+2)}{x}$

14. $f(x) = \dfrac{x^2 - 4}{x - 2}$

15. $f(x) = \dfrac{x + 2}{x^2 - 4}$

16. $f(x) = \dfrac{x - 3}{3x + 1}$